Random Sampling and Variation

POSC 3410 – Quantitative Methods in Political Science

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Goal for Today

Begin our discussion of inferential statistics with random sampling and variation.

Getting into Inferential Statistics

We are getting deeper into applied research design.

- We are no longer interested in just descriptive statistics.

We want to get into **inferential statistics.**

What is the Population?

Let’s be clear with our terms.

- **Population:** the universe of cases we want to describe.
- **Population parameter:** the unknown segment of the population we want to estimate.

In our past examples, this would be American attitudes on gun control or political tolerance (for example).

Sampling the Population

Big problem: we do not have data on approximately 300-million-plus Americans living in this country.

- Instead, we use a **sample**, a subset of cases drawn from the population.
- When done properly, the **sample statistic** gives us an estimate of the population parameter.

Instances in which we have the universe of all potential cases are called a **census**.

- Examples: Supreme Court decisions, wars in the modern state system.
- These create inferential issues of interest to Bayesians.

The Random Sample

The sampling concept is a foundation of applied political science.

- However, it is possible to do this incorrectly.
- The implications of a botched sampling process are far-reaching.

A proper sampling process is called a **random sample**.

- The random sample has no issues of **selection bias.**

A random sample with no selection bias guarantees no systematic error in the sample and no issue of bias in our inferences.

The 1936 Literary Digest Poll

Who would win the 1936 Presidential election? *The Literary Digest* wanted to know.

- It obtained names and addresses of all telephone users and car owners.
- It combined those names with the magazine's subscriber base to create a sample of 10 million people.
- *The Literary Digest* mailed sample ballots and received 2.4 million back.
- **Their conclusion:** Alf Landon would win in a landslide.

Actual conclusion: FDR won in a landslide. *The Literary Digest* went out of business a few years later.

The 1936 Literary Digest Poll

How did *The Literary Digest* get this so wrong? The answer lay in the **sampling frame**.

- Though such a sampling frame seems uncontroversial now, *The Literary Digest*'s sampling frame picked up mostly Landon supporters.

Mailing out ballots invited an additional issue of **response bias**.

- Those that *really* detested FDR were more likely to respond.

The 1936 Literary Digest Poll

All told, the sample statistic differed systematically from the true, underlying population parameter.

- This is textbook bias in inferential statistics.

How do we not repeat *The Literary Digest*'s error?

Obtaining a Random Sample

Professional polling firms like Gallup use random number generators.

- Every eligible unit of the population has a number applied to them.
- Your book mentions this with respect to a hypothetical university.
- A random number generator randomly draws *n* numbers from population.

Because the process is strictly random, any differences between the sample and those not sampled are also random.

Sampling Error

Eliminating bias *does not* eliminate error.

- Random sampling purposely introduces **random sampling error**.
- This may seem bad, but systematic error is always the greater evil.

We can actually estimate the random sampling error.

Understanding Random Sampling Error

The population parameter that interests us is defined as follows.

- Population parameter = Sample statistic + R.S.E.
- “R.S.E.” = random sampling error.

We consider two factors when we measure random sampling error.

1. The size of the sample
2. The amount of variation in the population parameter.

$$\text{R.S.E.} = \frac{\text{Variation component}}{\text{Sample size component}}$$

Understanding Random Sampling Error

Notice the fraction. As the sample size component increases, random sampling error decreases.

- However, the effect is non-linear.
- Sample size component is actually the square root of the number of observations in the sample.

$$\text{Sample size component} = \sqrt{n}$$

Sample Size

All else equal, an increase in sample size from 100 to 400 offers only a two-fold reduction in sampling error.

```
28/sqrt(100)
```

```
## [1] 2.8
```

```
28/sqrt(400)
```

```
## [1] 1.4
```

The implication: increase the sample size by as much as possible.

Sample Size

However, increasing the sample size may be costly.

- It may even incentivize non-random sampling.

This is why most surveys have a “sweet spot” between 1,000 and 3,000.

Sample Size

```
(28/sqrt(100))/(28/sqrt(1000))
```

```
## [1] 3.162278
```

```
(28/sqrt(1000))/(28/sqrt(10000))
```

```
## [1] 3.162278
```

Sample Variation

As the variation component increases, random sampling error increases.

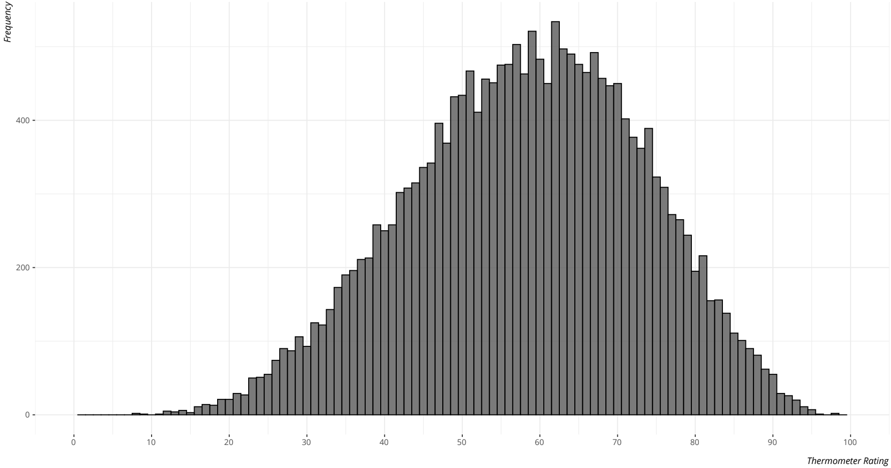
- We have a statistic for this variation: the **standard deviation**.

Compare the following two histograms

- Histogram 1: high spread, high standard deviation
- Histogram 2: low spread, low standard deviation

High Spread, High Variation

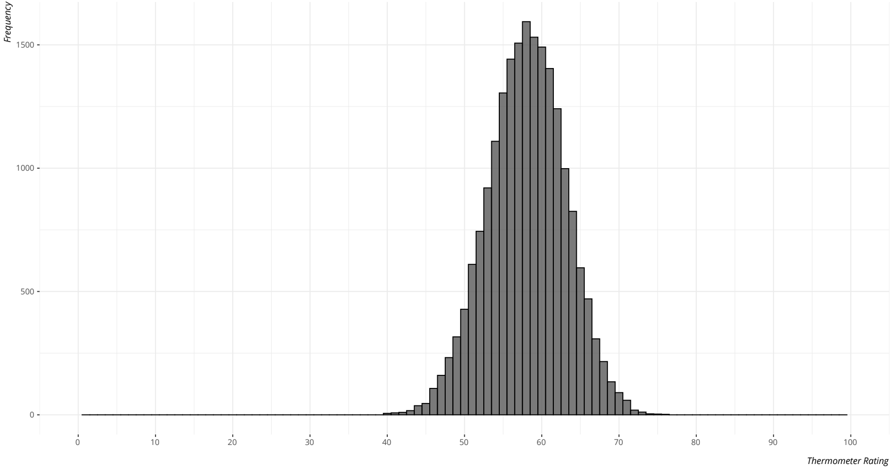
Data are simulated to be bounded normal (technically beta distribution) with a mean of 58 and a standard deviation of 15.



Data: hypothetical with set parameters fed to a bounded normal (really: beta) distribution.

Low Spread, Low Variation

Data are simulated to be bounded normal (technically beta distribution) with a mean of 58 and a standard deviation of 5.



Data: hypothetical with set parameters fed to a bounded normal (really: beta) distribution.

Calculating Standard Deviation

We can actually calculate this.

- Assume known population values, *N* and μ.

Important to know before going forward:

- *N* refers to number of cases in a population. *n* refers to sample size.
- μ refers to central tendency of population. $\bar{x}$ refers to sample mean.

Important heuristic: Greek letters refer to properties of the population, not the sample.

Calculating the Standard Deviation

1. Subtract μ from every value in the population.
2. Square that deviation for every observation.
    - If you didn't, the sum of deviations would equal zero.
3. Add those squared deviations together.
    - This is the sum of squared deviations.
4. Calculate arithmetic mean for sum of squared deviations.
    - This is an important statistic called the **variance**.
5. Take the square root of the variance.

Standard Error of a Sample Mean

Random sampling eliminates bias, but invites random error.

- We want to eliminate random error if we can.
- It's not as perilous as systematic error, but it's still unwanted noise.

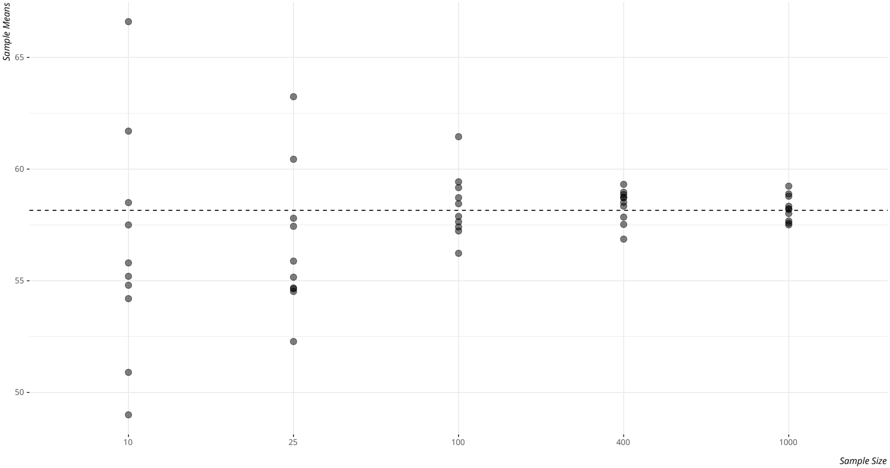
One sure way to do this is to increase the sample size.

- Consider the next two plots.
- μ is 58 in both panels, but standard deviation σ is greater in the first than the second.

Sometimes we can't do much about natural variation in the population, but we can increase the sample size to get more faithful sample statistics.

Ten Sample Means of Varying Sample Sizes from a High Variation Population

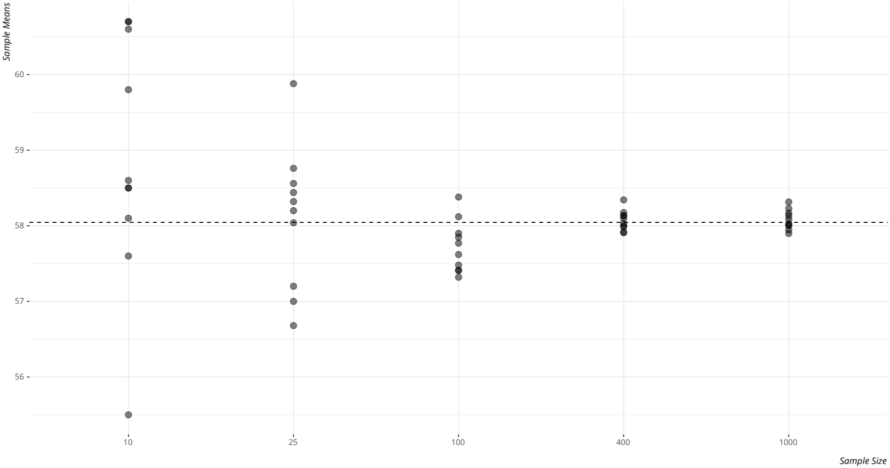
The diminishing returns of increasing sample size emerge around 400 observations, even as the spread in these simulated data is quite large.



Data: hypothetical with set parameters (mean: 58, sd: 15, n=20,000) fed to a bounded normal (really: beta) distribution.

Ten Sample Means of Varying Sample Sizes from a Low Variation Population

The diminishing returns of increasing sample size again emerge around 400 observations.



Data: hypothetical with set parameters (mean: 58, sd: 5, n=20,000) fed to a bounded normal (really: beta) distribution.

Standard Error of a Sample Mean

We have our full formula for calculating random sampling error.

$$\text{Standard error of sample mean} = \frac{\sigma}{\sqrt{n}}$$

Assume $\bar{x}$ = 59, σ = 24.8, *n* = 100.

- Standard error = 2.48
- We believe the population parameter is between 56.2 and 61.48
- Since we know μ = 58, we know that’s true.

Conclusion

We can obtain a reasonable estimate of a known population parameter by randomly sampling the population.

- The sample estimate becomes a good guess of the population parameter when we know μ a priori.

What we don't know yet:

- How likely is our $\bar{x}$ going to be within one standard error of μ?
- What if we don't know μ, but have an idea of what it could be?

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